Generation and Detection of Fock-States of the Radiation Field

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Z. Naturforsch. **56 a,** 117–123 (2001); received January 12, 2001

Presented at the 3rd Workshop on Mysteries, Puzzles and Paradoxes in Quantum Mechanics, Gargnano, Italy, September 17 - 23, 2000.

In this paper we give a survey of our experiments performed with the micromaser on the generation of Fock states. Three methods can be used for this purpose: the trapping states leading to Fock states in a continuous wave operation, state reduction of a pulsed pumping beam, and finally using a pulsed pumping beam to produce Fock states on demand where trapping states stabilize the photon number.

Key words: Quantum Optics; Cavity Quantum Electrodynamics; Nonclassical States; Fock States; One-Atom-Maser.

I. Introduction

The quantum treatment of the radiation field uses the number of photons in a particular mode to characterize the quantum states. In the ideal case the modes are defined by the boundary conditions of a cavity giving a discrete set of eigen-frequencies. The ground state of the quantum field is represented by the vacuum state consisting of field fluctuations with no residual energy. The states with fixed photon number are usually called Fock or number states. They are used as a basis in which any general radiation field state can be expressed. Fock states thus represent the most basic quantum states and differ maximally from what one would call a classical field. Although Fock states in analogous cases are routinely observed such as e. g. for the vibrational motion of ions in traps [1], Fock states of the radiation field are very fragile and very difficult to produce and maintain. They are perfectly number-squeezed, extreme sub-Poissonian states in which intensity fluctuations vanish completely. In order to generate these states it is necessary that the mode considered has minimal losses and the thermal field, always present at finite temperatures, has to be eliminated to a large extent since it causes photon number fluctuations.

The one-atom maser or micromaser [2] is the ideal system to realize Fock states. In the micromaser highly excited Rydberg atoms interact with a single mode of a superconducting cavity which can have a quality factor as high as 4×10^{10}, leading to a photon lifetime in the cavity of 0.3 s. The steady-state field generated in the cavity has already been the object of detailed studies of the sub-Poissonian statistical distribution of the field [3], the quantum dynamics of the atom-field photon exchange represented in the collapse and revivals of the Rabi nutation [4], atomic interference [5], bistability and quantum jumps of the field [6], atom-field and atom-atom entanglement [7]. The cavity is operated at a temperature of 0.2 K leading to a thermal field of about 5×10^{-2} photons per mode.

There have been several experiments published in which the strong coupling between atoms and a single cavity mode is exploited (see e. g. [8]). The setup described here is the only one where maser action can be observed and the maser field investigated. In our setup the threshold for maser action is as small as 1.5 atoms/s. This is a consequence of the high value of the quality factor of the cavity which is three orders of magnitude larger than that of other experiments with Rydberg atoms and cavities [9].

In this paper we present three methods of creating number states in the micromaser. The first is by way of the well known trapping states, which are generated in a c. w. operation of the pumping beam and lead

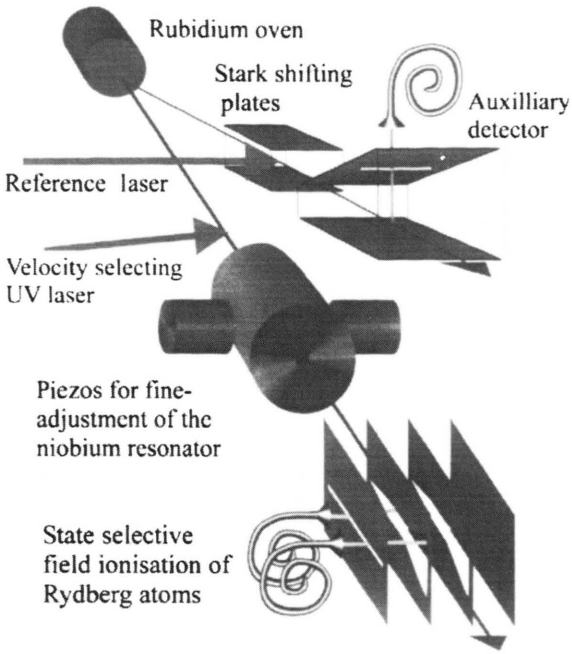

Fig. 1. The micromaser setup. For details see [9].

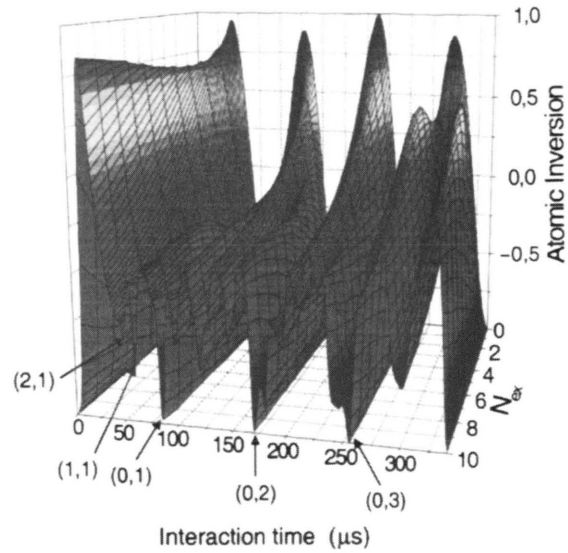

Fig. 2. A theoretical plot, in which the trapping states can be seen as valleys in the N_{ex} direction. As the pump rate is increased, the formation of the trapped states from the vacuum can be seen. The positions of trapping states are indicated by arrows with the respective designation (n, k).

to Fock states with high purity. We also present a second method where the field is prepared by state reduction and the purity of the states generated is investigated by a probing atom. It turns out that the two methods of preparation of Fock states lead to a similar result for the purity of the Fock states. The third method pumps the cavity with a pulsed beam using the trapping condition to stabilize the photon number in our cavity. This method produces Fock states on demand.

II. The One-Atom Maser and the Generation of Fock-States using Trapping States

The one-atom maser or micromaser is the experimental realisation of the Jaynes-Cummings model [10], as it allows to study the interaction of a single atom with a single mode of a high Q cavity. The setup used for the experiments is shown in Fig. 1 and has been described in detail in [11]. Briefly, in this experiment a ^{3}He-^{4}He dilution refrigerator houses the microwave cavity which is a closed superconducting niobium cavity. A rubidium oven provides two collimated atomic beams: a central one passing directly into the cryostat and a second one directed to an additional excitation region. The second beam was used as a frequency reference. A frequency doubled dye laser (λ = 294 nm) was used to excite rubidium (^{85}Rb) atoms to the 63 $P_{3/2}$ Rydberg state from the 5 $S_{1/2}$ ($F = 3$) ground state.

Velocity selection is provided by angling the excitation laser towards the main atomic beam at about 11° to the normal. The dye laser was locked, using an external computer control, to the 5 $S_{1/2}$ ($F = 3$) – 63 $P_{3/2}$ transition of the reference atomic beam excited under normal incidence. The reference transition was detuned by Stark shifting the resonance frequency using a stabilized power supply. This enabled the laser to be tuned while remaining locked to an atomic transition. The maser frequency corresponds to the transition between 63 $P_{3/2}$ and 61 $D_{5/2}$. The Rydberg atoms are detected by field ionization in two detectors set at different voltages, so that the upper and lower states of the maser transition can be investigated separately.

The trapping states are a steady-state feature of the maser field peaked in a single photon number; they occur in the micromaser as a direct consequence of field quantisation. At low cavity temperatures the number of blackbody photons in the cavity mode is reduced and trapping states begin to appear [11, 12]. They occur when the atom field coupling constant

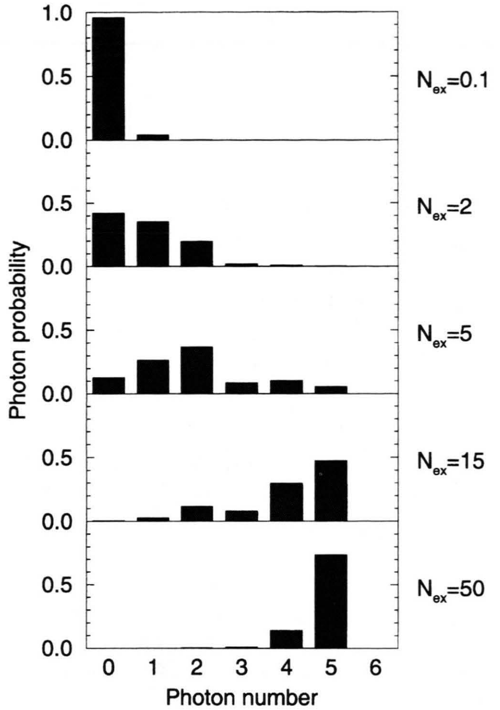

Fig. 3. A numerical simulation of the photon number distribution as the atomic pump rate (N_{ex}) is increased until the cavity field is in a Fock state with a high probability. Shown is the example of the $n = 5$ Fock state. A further increase of N_{ex} beyond 50 does not change the distribution.

given by the Rabi frequency Ω, and the interaction time, t_{int}, are chosen such that in a cavity field with n photons each atom undergoes an integer number, k, of Rabi cycles. This is summarised by the condition

$$\Omega t_{int}\sqrt{n+1} = k\pi . \quad (1)$$

When (1) is fulfilled the cavity photon number is left unchanged after the interaction of an atom and hence the photon number is "trapped". This will occur regardless of the atomic pump rate N_{ex}, where N_{ex} is the rate of pumping atoms in the excited state per decay time of the cavity. The trapping state is therefore characterised by the photon number n and the number of integer multiples of full Rabi cycles k.

The build-up of the cavity field can be seen in Fig. 2, where the emerging atom inversion $I = P_g - P_e$ is plotted against interaction time and pump rate; $P_{g(e)}$ is the probability of finding a ground (excited) state atom. At low atomic pump rates (low N_{ex}) the maser field cannot build up and the maser exhibits Rabi

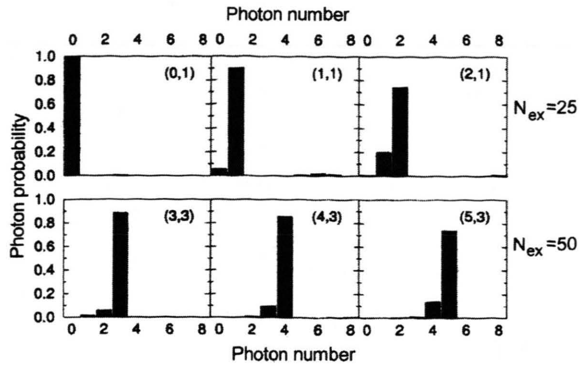

Fig. 4. In simulations of the maser operation, Fock states from $n = 0$ to $n = 5$ can be readily generated for achievable experimental conditions ($n_{th} = 10^{-4}$).

oscillations due to the interaction with the vacuum field. At the positions of the trapping states, the field increases until it reaches the trapping state condition. This manifests itself as a reduced emission probability and hence as a dip in the atomic inversion. Once in a trapping state the maser will remain there regardless of the pump rate. The trapping states show up therefore as valleys in the N_{ex} direction. Figure 3 shows the photon number distribution as the pump rate is increased for the special condition of the five photon trapping state. The photon distribution develops from a thermal distribution towards higher photon numbers until the pump rate is high enough for the atomic emission to be stabilized by the trapping state condition. As the pump rate is further increased, and in the limit of a low thermal photon number, the field continues to build up to a single trapped photon number and the steady-state distribution approaches a Fock state.

Owing to blackbody radiation at finite temperatures, there is always a small probability of having a thermal photon enter the mode. The presence of a thermal photon in the cavity disturbs the trapping state condition and an atom can emit a photon. This causes the field to change around the trapping condition.

Note that under readily achievable experimental conditions it is possible for the steady-state field in the cavity to approach a Fock state with a high fidelity. Under the present experimental conditions the main deviation from a pure Fock state results from dissipation of the field in the cavity. If a photon disappears it takes a little while until the next incoming excited atom can be used to replace the lost photon. Therefore smaller photon numbers show up besides the considered Fock state. Figure 4 shows micromaser simulations for achievable experimental conditions, in which Fock states with high purity are created from $n = 0$ to $n = 5$. The experimental realisation requires a pump rate of $N_{ex} = 50$, a temperature of about 100 mK [6], a high selectivity of atomic velocity and very low mechanical noise of the system [11, 13]. The details of the production of Fock states using the trapping condition are described in [11].

III. Dynamical Preparation of Number States in a Cavity

When the atoms leave the cavity in a micromaser experiment they are in an entangled state with the field. A method of state reduction was suggested by Krause et al. [14] to observe the build up of the cavity field to a known Fock state. State reduction uses the entanglement produced by the interaction of an atom with a cavity field to project the field onto a well defined number state. If the field is in an initial state $|n\rangle$ then an interaction of an atom with the cavity leaves the cavity field in a superposition of the states $|n\rangle$ and $|n+1\rangle$ and the atom in a superposition of the internal atomic states $|e\rangle$ and $|g\rangle$.

$$\Psi = \cos(\phi)|e\rangle|n\rangle - i\sin(\phi)|g\rangle|n+1\rangle, \tag{2}$$

where ϕ is an arbitrary phase. The state selective field ionisation measurement of the internal atomic state reduces the field to one of the states $|n\rangle$ or $|n+1\rangle$. State reduction is independent of the interaction time, hence a ground state atom always projects the field onto the $|n+1\rangle$ state independent of the time spent in the cavity. This results in an *a priori* probability of the maser field being in a specific but unknown number state [14]. If the initial state of the cavity is the vacuum, $|0\rangle$, then a number state created is equal to the number of ground state atoms that were collected within a suitably small fraction of the cavity decay time.

In a system governed by the Jaynes-Cummings Hamiltonian, spontaneous emission is reversible and an atom in the presence of a resonant quantum field undergoes Rabi oscillations. That is the relative populations of the excited and ground states of the atom oscillate at a frequency $\Omega\sqrt{n+1}$, where Ω is the atom field coupling constant. Experimentally we measure the atomic inversion. In the presence of dissipation a fixed photon number n in a particular mode is not observed and the field always evolves into a mixture of such states. Therefore the inversion is generally given by

$$I(n, t_{int}) = -c \sum_n P_n \cos(2\Omega\sqrt{n+1}\,t_{int}), \tag{3}$$

where P_n is the probability of finding n photons in the mode and t_{int} the interaction time of the atoms with the cavity field. The factor c considers the reduction of the signal amplitude as a result of dark counts.

The experimental verification of the presence of Fock states in the cavity corresponds to a pump-probe experiment in which a pump atom prepares a quantum state in the cavity and the Rabi phase of the emerging probe atom measures the quantum state. The signature that the quantum state of interest has been prepared is simply the detection of a defined number of ground state atoms. To verify that the correct quantum state has been projected onto the cavity, a probe atom is sent into the cavity with a variable, but well defined interaction time. As the formation of the quantum state is independent of interaction time we need not to change the relative velocity of the pump and probe atoms, thus reducing the complexity of the experiment. In this sense we are performing a reconstruction of a quantum state in the cavity using a similar method to that described by Bardoff et al. [15]. This experiment reveals the maximum amount of information that can be found relating to the cavity photon number. We have recently used this method to demonstrate the existence of Fock states up to n=2 in the cavity [16].

When the interaction time corresponding to the trapping state condition is met in this experiment, the formation of the cavity field is identical to that which occurs in the steady-state, hence the probe atom should perform an integer number of Rabi cycles. In fact this was observed experimentally [16], which indicates that the pulsed experiment is actually the formation stage of the steady-state experiment. One would therefore expect that the measured photon number distribution, in the dynamical measurement, would be the same as that predicted for the trapping states. State reduction is simply a method of observation that determines the appropriate moment for a measurement. In this sense the observation of a lower emission probability in the steady state is also a field-state measurement as the dip in the steady-state inversion measurement occurs for practically the same conditions as for the dynamical measurements described here [16].

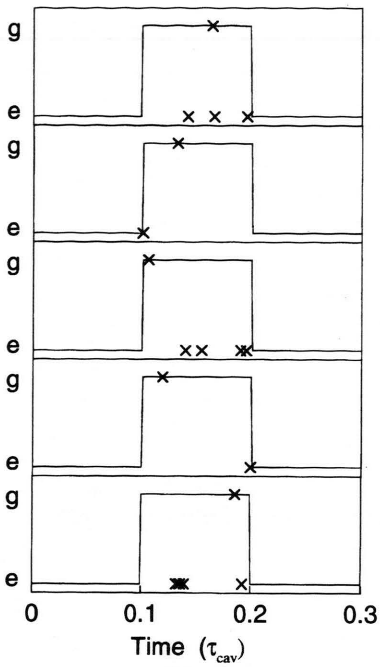

Fig. 5. A computer simulation of five sequential excitation pulses for Rydberg atoms. The interaction time is tuned to the (1, 1) trapping state. The cavity is initially in the vacuum state. Rydberg atoms are marked by crosses; those in the upper or lower maser level are marked on the lower and upper row respectively. For high pump rates, every pulse contains at least one lower-state atom with a deviation occurring once in every 50 pulses. The parameters used are: $\tau_{cav} = 100$ ms, $n_{th} = 0.03$, $\tau_{pulse} = 10$ ms, $N_{ex} = 40$ or $N_a = 4$ atoms per pulse.

IV. Preparation of Fock States on Demand

In the following we describe another variant of a dynamical Fock-state preparation with the micromaser [17]. To demonstrate the principle of the source described here, Fig. 5 shows the simulation of a sequence of five arbitrary atom pulses using a Monte Carlo calculation in which the micromaser is operated in the (1, 1) trapping state. In each pulse there is a single emission event, producing a single lower state atom and leaving a single photon in the cavity. The atom-cavity system is then in the trapping condition; as a consequence the emission probability is reduced to zero and the photon number is stabilized. In steady state operation, the influence of thermal photons and variations in interaction time or cavity tuning complicates this picture, resulting in deviations from Fock states [18]. Pulsed excitation, however, reduces the influence of such

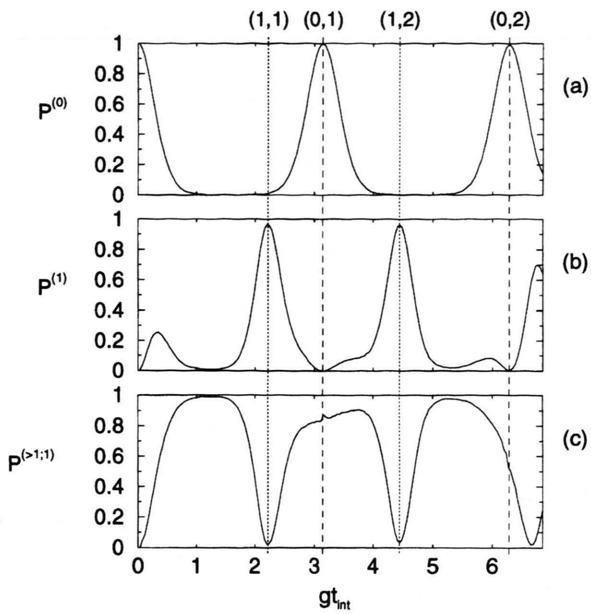

Fig. 6. The figure presents the probability of finding: (a) no lower-state atoms per pulse $P^{(0)}$, (b) exactly one lower-state atom per pulse $P^{(1)}$, and (c) of finding a second lower-state atom if one has already been detected $P^{(>1;1)}$. Parameters for these simulations were $\tau_{pulse}=0.02\tau_{cav}$, $N_a = 7$ atoms and $n_{th} = 10^{-4}$. The maximum value of $P^{(1)}$ is 98% for the (1, 1) trapping state.

effects and the generated Fock states show a high purity.

Figure 6 presents three curves, obtained from the computer simulation, that illustrate the behaviour of the maser under pulsed excitation as a function of interaction time for more ideal (but achievable) experimental parameters. The simulations show the probability of finding: no lower-state atom per pulse ($P^{(0)}$), exactly one lower-state atom per pulse ($P^{(1)}$), and the conditional probability of finding a second lower-state atom in a pulse already containing one ($P^{(>1;1)}$). The latter plot of the conditional probability, $P^{(>1;1)}$, is relatively insensitive to the absolute values of the atomic detection efficiency and therefore has advantages when comparisons with experimental data are performed [17].

It follows from Fig. 6 that, with an interaction time corresponding to the (1, 1) trapping state, both one photon in the cavity and a single atom in the lower-state are produced with 98% probability. In order to maintain an experimentally verifiable quantity, the simulations presented here relate to the production of lower-state atoms rather than to the Fock state left in the cavity. However, pulse lengths are rather short ($0.01\tau_{cav} \leq \tau_{pulse} \leq 0.1\tau_{cav}$), so there is little dissipation and the probability of finding a one photon state in the cavity following the pulse is very close to the probability of finding an atom in the lower-state. Atomic beam densities must also be chosen with care to avoid short pulses with high atom density that could violate the one-atom-at-a-time condition.

Note that at no time in this process is a detector event required to project the field. The field evolves to the target photon number state, when a suitable interaction time has been chosen so that the trapping condition is fulfilled.

It should be noted that for thermal photon numbers as high as $n_{th} = 0.1$ or for t_{int} fluctuations of up to 10% (both beyond the current experimental parameters), simulations show that Fock states are still prepared with an 80-90% fidelity. This is considerably better than for steady state trapping states, where highly stable conditions with low thermal photon numbers are required [12, 11, 18].

The present setup of the micromaser was specifically designed for steady state operation. Nevertheless the current apparatus does permit a comparison between theory and experiment in a relatively small parameter range.

A method was used for the comparison with the experiment which is described briefly in the following. During the interaction, strong coupling between pumping atoms and the cavity field creates entanglement between internal atomic levels and the cavity field. Subsequent pumping atoms will therefore also become entangled both with the field and a previous pumping atom. The correlations between subsequent atomic levels are determined by the dynamics of the atom-cavity interaction. The connection between population correlations and the micromaser dynamics has been studied in detail in [7, 19]. It is important to note that even in the presence of lost counts, the correlations between subsequently detected atoms are maintained. Thus, rather than a measurement of the single-atom-per-pulse probability $P^{(1)}$ which is heavily dependent on exact knowledge of detection efficiency, it is more useful to measure atom pair correlations given by $P^{(>1;1)}$, owing to the insensitivity of the parameter to detector efficiencies. Experimentally the parameter is obtained via

$$P^{(>1;1)} = \frac{N_{gg}}{N_{gg} + N_{eg} + N_{ge}}, \tag{4}$$

where for example N_{eg} is the probability of detecting a pair of atoms containing first an upper state atom (e) and then a lower-state atom (g) within a pulse. Equation (4) provides both a value appropriate to the existent correlation and is directly related to the total probability of finding one atom per pulse. Although $P^{(>1;1)}$ is insensitive to the absolute detector efficiency, it does depend on the relative detector efficiencies (which are nearly equal) and the miscount probability (the probability that a given atomic level is detected in the wrong detector). Each has been measured experimentally.

The source presented here has the significant advantage over our previous method of Fock state creation [16] of being unconditional and therefore significantly faster in preparing a target quantum state. Previously, state reduction by detection of a predefined number of lower state atoms was used to prepare the state with 95% fidelity. However this method has the disadvantage that it is affected by non perfect detectors. In the current experiment, however, the cavity field is correctly prepared in 83.2% of the pulses and is independent of any detector efficiencies. Improving the experimental parameters we can expect to reach conditions for which 98% of the pulses prepare single photon Fock states and a single atom in the lower-state.

VI. An Application of Trapping States – the Generation of GHZ States

The following proposal for the creation of states of the Greenberger-Horne-Zeilinger (GHZ) type[20] is an application of the vacuum trapping state. For a review on the generation of atom-atom entanglement in a micromaser see [7]. For the present proposal we assume that the cavity is initially empty, and two excited atoms traverse it consecutively. The velocity of the first atom, and its consequent interaction time, is such that it emits a photon with the probability $(\sin\varphi_1)^2 = 51.8\%$, where $\varphi_1 = 0.744\pi$ is the corresponding Rabi angle ϕ of (2) for $n = 0$. The second atom arrives with the velocity dictated by the vacuum trapping condition; for $n = 0$ it has $\phi = \pi$ in (2), so that $\phi = \sqrt{2}\,\pi$ for $n = 1$. Assuming that the duration of the whole process is short on the scale set by the lifetime of the photon, we thus have

$$|0, e, e\rangle \xrightarrow[\text{atom}]{\text{first}} |0, e, e\rangle \cos(\varphi_1) - i|1, g, e\rangle \sin(\varphi_1) \tag{5}$$

$$|0,e,e\rangle \xrightarrow[\text{atom}]{\text{second}} -|0,e,e\rangle\cos(\varphi_1) \\ -|2,g,g\rangle\sin(\varphi_1)\sin(\sqrt{2}\,\pi) \\ -i|1,g,e\rangle\sin(\varphi_1)\cos(\sqrt{2}\,\pi),$$

where, for example, $|1,g,e\rangle$ stands for "one photon in the cavity & first atom in the ground state & second atom excited." With the above choice of $\sin(\varphi_1) = 0.720$, we have $\cos(\varphi_1) = \sin(\varphi_1)\sin(\sqrt{2}\,\pi) = -0.694$, so that the two components with even photon number ($n = 0$ or $n = 2$) carry equal weight and occur with a joint probability of 96.3%. The small 3.7% admixture of the $n = 1$ component can be removed by measuring the parity of the photon state [21] and conditioning the experiment to even parity. The two atoms and the cavity field are then prepared in the entangled state

$$\Psi_{\mathrm{GHZ}} = \frac{1}{\sqrt{2}}\Big(|0,e,e\rangle + |2,g,g\rangle\Big), \tag{6}$$

which is a GHZ state of the Mermin kind [22] in all respects. For a detailed discussion of the method described here see [23].

VI. Conclusion

In this paper we gave a survey of the possibilities for generating Fock states in the micromaser. The generation of Fock states on demand has been experimentally confirmed and will be published elsewhere [17]. The possibility to generate Fock states will allow us to perform the reconstruction of a single photon field or other Fock states in a next step.

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